Estimation of Power Distribution in VLSI Interconnects

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Introduction

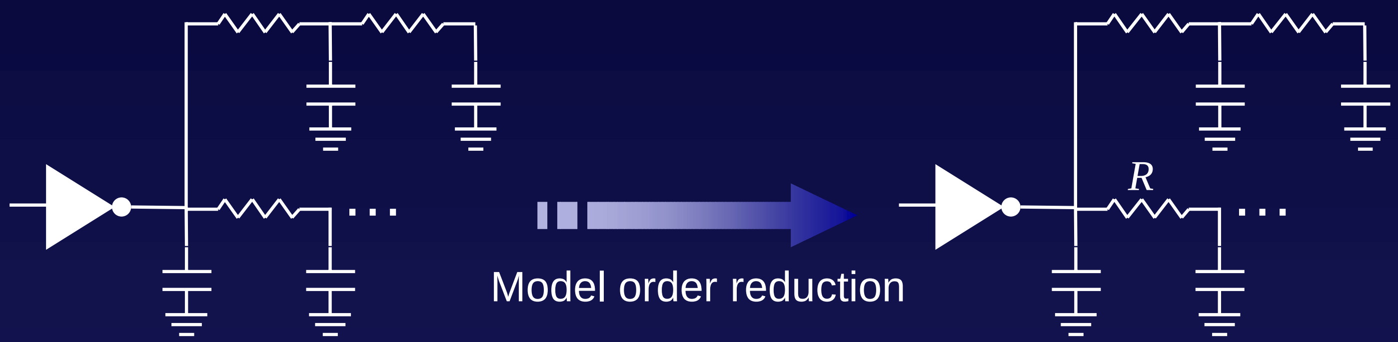
- **Scaling in VLSI**
  - Decreasing gate length, gate oxide, supply voltage
  - Increasing speed, cost-performance
- **Unfavorable effects due to VLSI scaling**
  - Increasing **power** density
  - **Complexity** of system and design
  - **Interconnect** related issues: delay, current density, noise

Introduction

- **Deep submicron interconnects**
  - Decreasing metal pitch
  - Increasing aspect ratio
  - Increasing metallization levels
  - Increasing line resistance and wire-to-wire capacitance
- **Problems and issues**
  - Smaller geometry and denser pattern: RC delay, signal integrity, crosstalk noise, delay fluctuation
  - Larger current: IR drop and **reliability** (electromigration)

Introduction

- **Reliability problem**
  - Current density in metal lines increases
  - Temperature of interconnect increases
  - MTF (Mean Time to Failure) decreases
- **Problem of power distribution estimation**

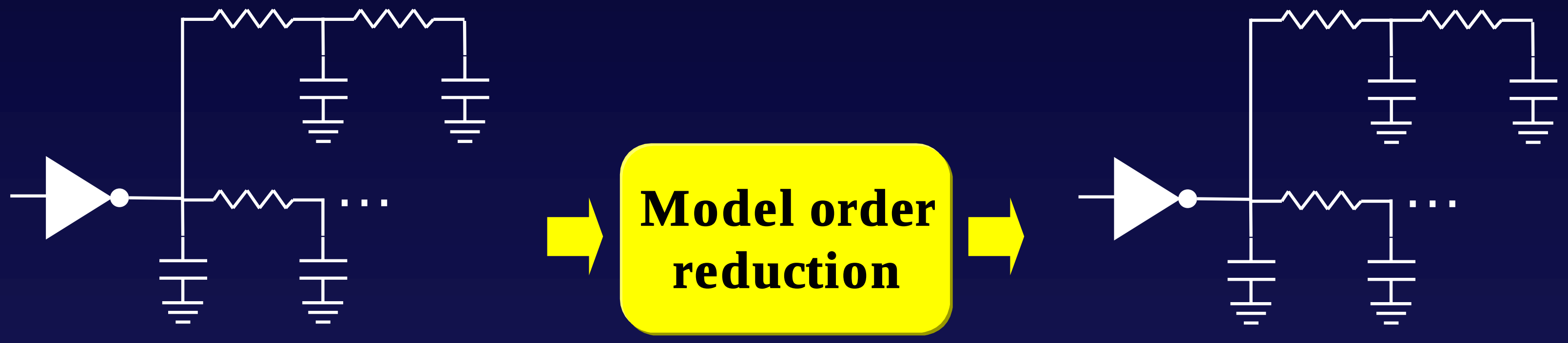


$$J(s) = \frac{b(s-k_1)}{(s-p_1)(s-p_2)}$$

$$E = R\int_0^T j^2(t)\,dt$$

Model Order Reduction

- **Model order reduction**
  - Reduce the circuit to a smaller representation consisting of dominant poles from the original circuit

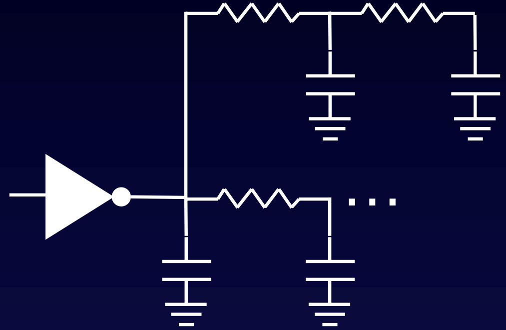


$$H(s) = \frac{a(s-k_1)(s-k_2)\cdots(s-k_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$H(s) = \frac{b(s-k_1)}{(s-p_1)(s-p_2)}$$

Model Order Reduction

- **Moment matching-based**



$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u, \quad s\mathbf{X} = \mathbf{A}\mathbf{X} + \mathbf{b}\mathrm{U},$$

$$y = \mathbf{c}^T\mathbf{x} \qquad Y = \mathbf{c}^T\mathbf{X}$$

$$H(s) = \mathbf{c}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} = \sum_{i=0}^{\infty} m_i s^i$$

$$m_i = -\mathbf{c}^T\mathbf{A}^{-i-1}\mathbf{b},$$

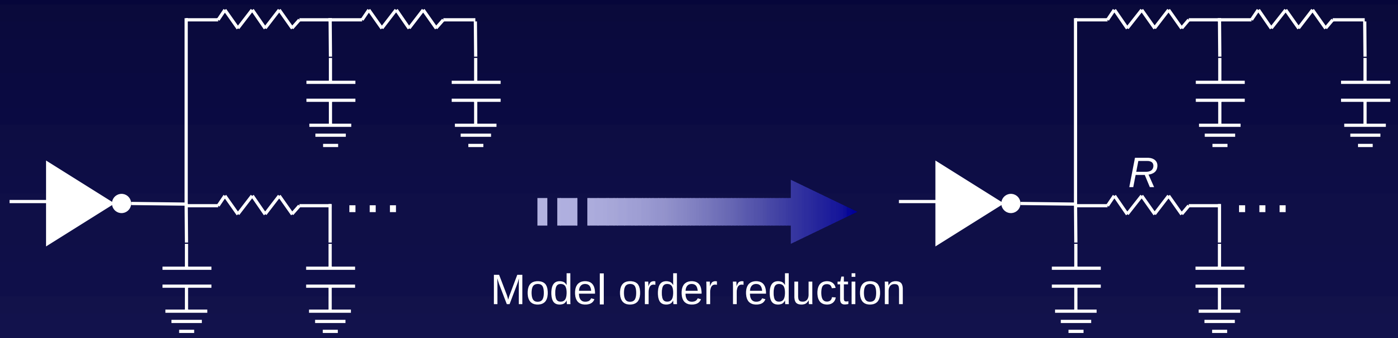
$$m_{i+1} = \mathbf{A}^{-1}m_i$$

$$\hat{H}(s) = \frac{n_{q-1}s^{q-1} + n_{q-2}s^{q-2} + \cdots + n_1 s + n_0}{s^q + d_{q-1}s^{q-1} + \cdots + d_1 s + d_0} = m_0 + m_1 s + \cdots + m_{2q-1}s^{2q-1}$$

$$\hat{H}(s) = \sum_{i=1}^{q} \frac{r_i}{s - p_i} \Longleftrightarrow \hat{h}(t) = \sum_{i=1}^{q} r_i e^{p_i t}$$

Power Distribution Estimation

- **Power distribution estimation of interconnect**
  - Given a linear(ized) RLC circuits
  - Find power consumption of each resistor branch of interconnect



$$J(s) = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

$$j(t) = r_1 e^{p_1 t} + r_2 e^{p_2 t}$$

$$E = R \int_0^T j^2(t)\, dt$$

Power Distribution Estimation

- **Definition of problem**
  - Given a reduced-order model of current at each resistor branch $J(s)=\sum_{i=1}^{q}\frac{r_i}{s-p_i}$
  - Derive $E=R\int_0^{\infty} j^2(t)dt$
- **Theorem 1**
  - If the Laplace transform of a time-domain signal $j(t)$, denoted by $J(s)$, has q singularities in the left half of the s-plane,

$$\int_0^{\infty} j^2(t)dt=\sum_{i=1}^{q} r_i$$

r_i: residue of $J(-s)J(s)$ at the singularity of $J(s)$

Power Distribution Estimation

- **Sketch of proof**

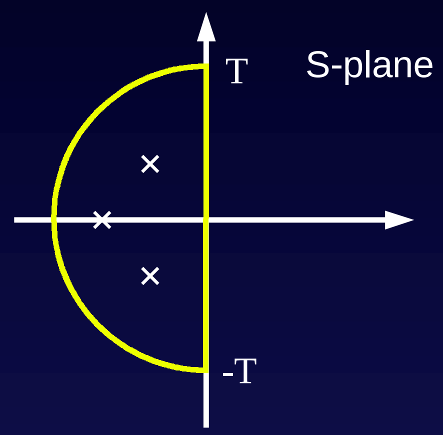
$$\int_0^\infty j^2(t)dt = \left[\int_0^\infty j^2(t)e^{-st}dt\right]_{s=0}$$

$$= \left[\mathrm{L}\{j(t)j(t)\}\right]_{s=0}$$

$$= \left[\frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} J(s-\omega)\,J(\omega)d\omega\right]_{s=0}$$

$$= \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} J(-\omega)\,J(\omega)d\omega$$

$$= \sum_{\text{singularity of } J(s)} \left(\text{residue of } J(-s)J(s)\right)$$



Power Distribution Estimation

- **Example**

$$J(s)=\frac{s+3}{(s+1)^2}=\frac{2}{(s+1)^2}+\frac{1}{s+1} \quad \longleftrightarrow \quad j(t)=2te^{-t}+e^{-t}$$

$$J(-s)J(s)=\frac{(-s+3)(s+3)}{(s-1)^2(s+1)^2}=\frac{2}{(s-1)^2}-\frac{\frac{5}{2}}{s-1}+\frac{2}{(s+1)^2}+\frac{\frac{5}{2}}{s+1}$$

$$\int_0^\infty j^2(t)dt=\int_0^\infty (2te^{-t}+e^{-t})^2 dt=\frac{5}{2}$$

Power Distribution Estimation

- **Theorem 2**
  - If the Laplace transform of a time-domain signal $j(t)$, denoted by $J(s)$, has q **simple poles** in the left half of the s-plane,

$$\int_0^\infty j^2(t)dt = \sum_{i=1}^{q} r_i J(-p_i)$$

r_i: residue of $J(s)$ at the pole p_i of $J(s)$

Power Distribution Estimation

- **Example**

$$J(s)=\frac{3s+5}{s^2+3s+2}=\frac{2}{s+1}+\frac{1}{s+2} \Longleftrightarrow j(t)=2e^{-t}+e^{-2t}$$

$$r_1J(-p_1)+r_2J(-p_2)=2(1+\frac{1}{3})+1(\frac{2}{3}+\frac{1}{4})=\frac{43}{12}$$

$$\int_0^\infty j^2(t)dt=\int_0^\infty (2e^{-t}+e^{-2t})^2 dt=\frac{43}{12}$$

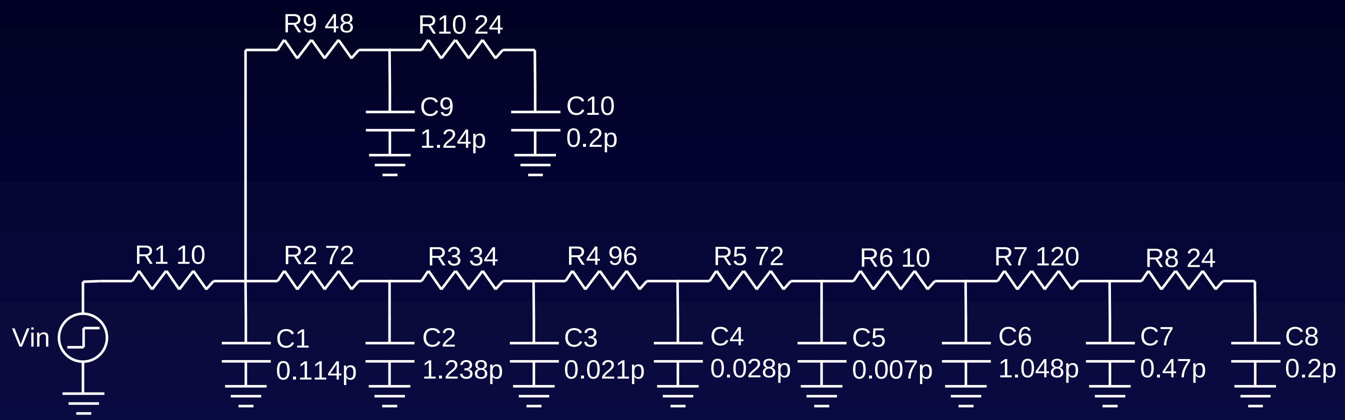
Experimental Results

- **Prototype tool**
  - SPICE-in and power-out
  - Moment matching-based model order reduction
- **Estimation accuracy**
  - Source of error: area under the **square** of j(t)
  - Comparison with SPICE

$$E = R \int_0^T j^2(t)\, dt$$

Experimental Results

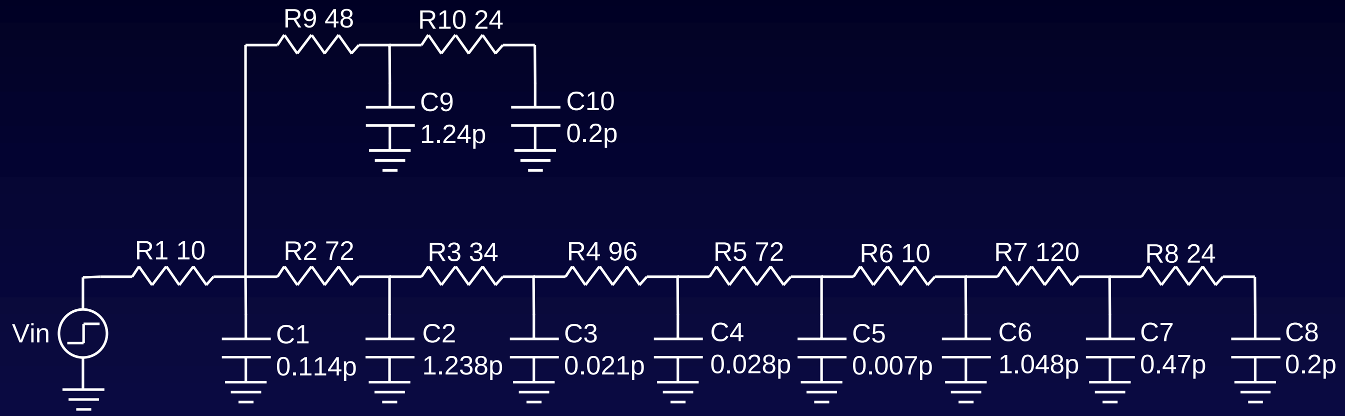
- **Numerical example**



Resistor	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	Avg. error	Max. error
SPICE	5.12	8.42	0.88	2.42	1.76	0.24	0.43	0.01	5.54	0.05		
1-pole	3.12	7.18	0.89	2.43	1.76	0.24	0.41	0.01	4.69	0.04	9.4%	39.1%
2-poles	4.81	8.39	0.88	2.42	1.76	0.24	0.44	0.01	5.53	0.05	1.2%	5.9%
3-poles	4.96	8.38	0.88	2.42	1.76	0.24	0.43	0.01	5.50	0.05	0.5%	3.2%

Experimental Results

- **Numerical example**



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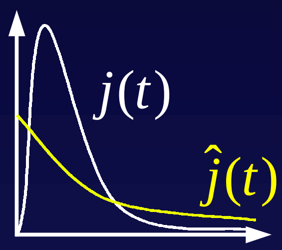
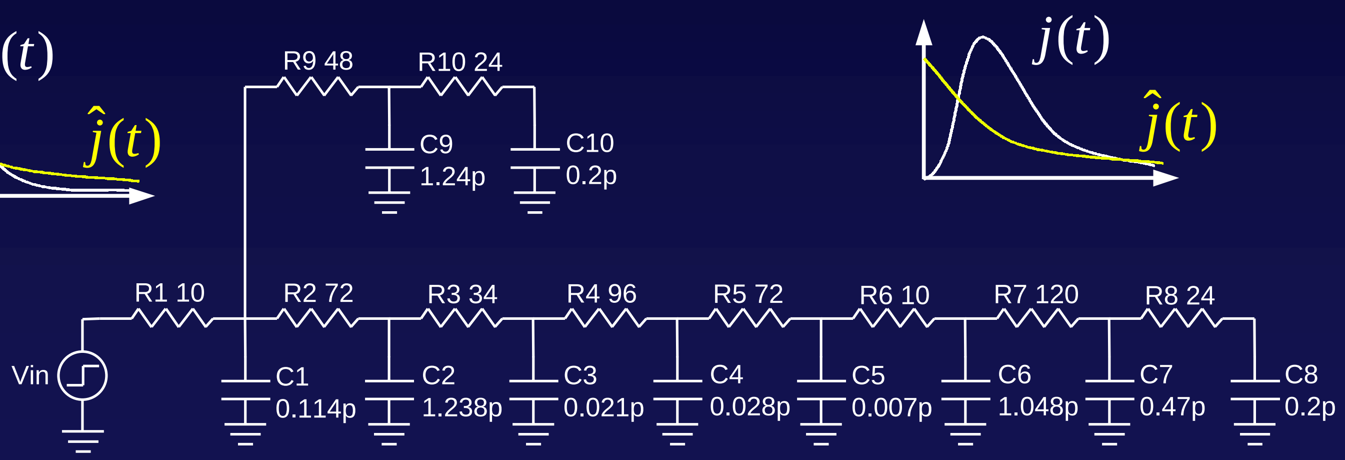
2-poles	4.81	8.39	0.88	2.42	1.76	0.24	0.44	0.01	5.53	0.05	1.2%	5.9%
3-poles	4.96	8.38	0.88	2.42	1.76	0.24	0.43	0.01	5.50	0.05	0.5%	3.2%

Experimental Results

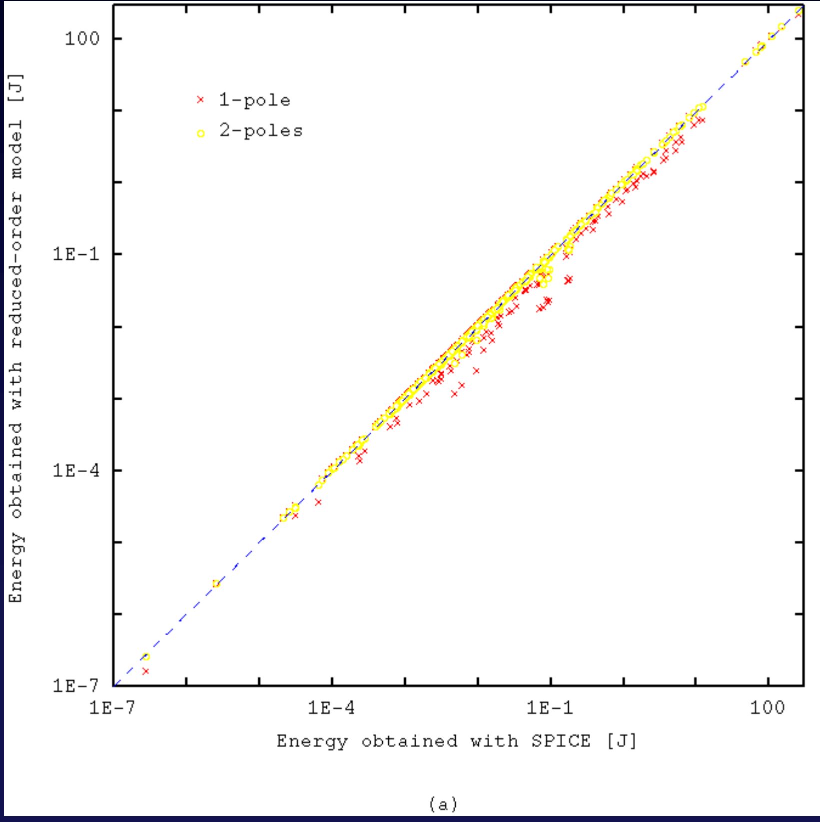
- **1-pole approximation**
  - Area under $j(t)$ and $\hat{j}(t)$ is the same
  - Area under $j^2(t)$ and $\hat{j}^2(t)$ depends on peakness and skewness of $j(t)$



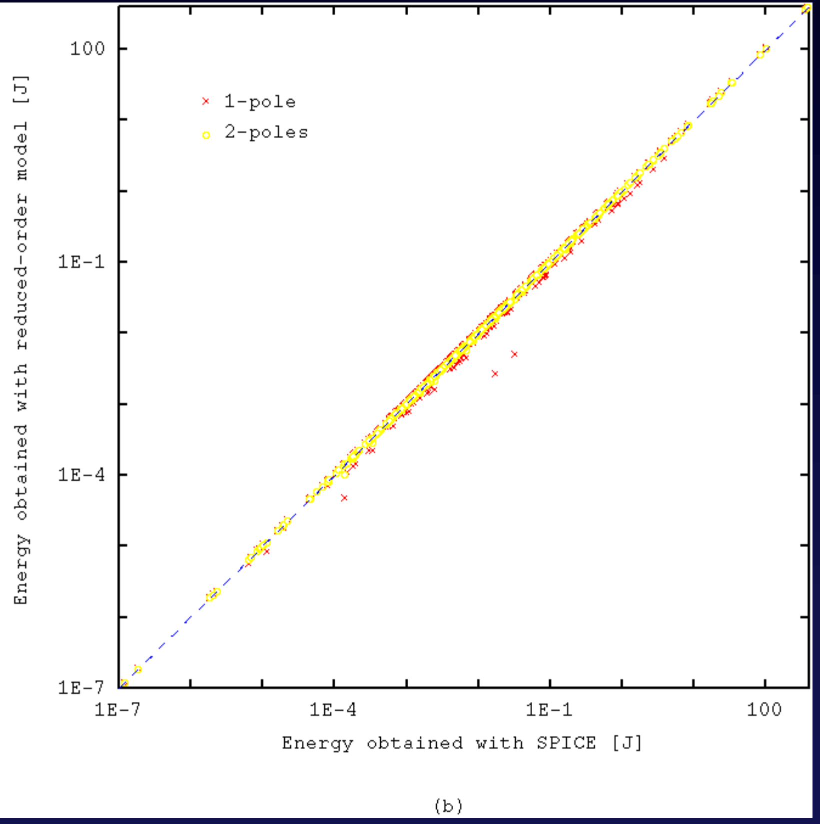


Experimental Results

- **Randomly-generated circuits**



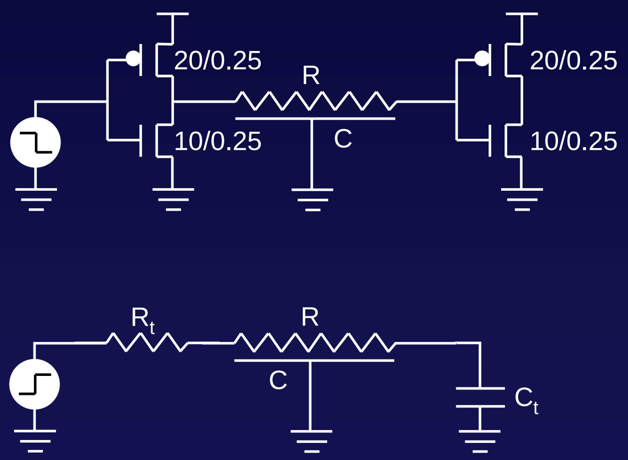
300 nodes

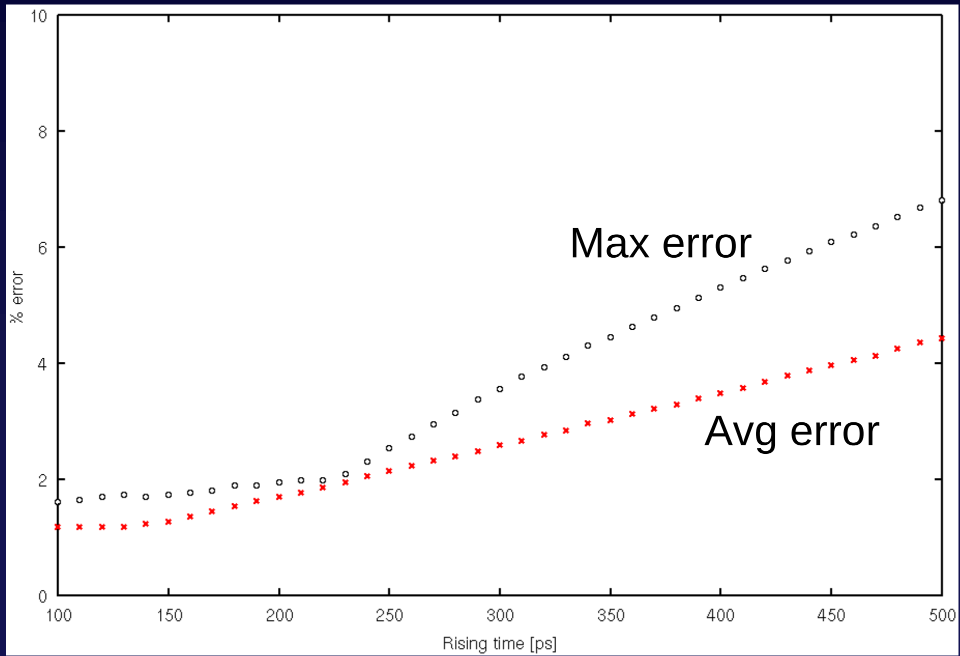


500 nodes

Driver Modeling

- **Verify simple linear region resistance approximation for power distribution estimation**
- **Well below 10% both for max and avg error**





Conclusion

- **Power distribution is important for deep submicron interconnects**
- **Establish theoretical background for power distribution analysis of VLSI interconnects**
- **Develop and verify a simple driver model**
- **Future work**
  - Fast yet accurate method
  - Investigation of accurate driver model