Estimation of Power Distribution in VLSI Interconnects

Youngsoo Shin and Takayasu Sakurai

Center for Collaborative Research
Univ. of Tokyo, Japan
Introduction

- **Scaling in VLSI**
  - Decreasing gate length, gate oxide, supply voltage
  - Increasing speed, cost-performance
- **Unfavorable effects due to VLSI scaling**
  - Increasing *power* density
  - *Complexity* of system and design
  - *Interconnect* related issues: delay, current density, noise
Introduction

- Deep submicron interconnects
  - Decreasing metal pitch
  - Increasing aspect ratio
  - Increasing metallization levels
  - Increasing line resistance and wire-to-wire capacitance

- Problems and issues
  - Smaller geometry and denser pattern: RC delay, signal integrity, crosstalk noise, delay fluctuation
  - Larger current: IR drop and reliability (electromigration)
Introduction

- Reliability problem
  - Current density in metal lines increases
  - Temperature of interconnect increases
  - MTF (Mean Time to Failure) decreases

- Problem of power distribution estimation

\[ E = R \int_0^T j^2(t) \, dt \]

\[ J(s) = \frac{b(s-k_1)}{(s-p_1)(s-p_2)} \]
**Model Order Reduction**

- **Model order reduction**
  - Reduce the circuit to a smaller representation consisting of dominant poles from the original circuit

\[
H(s) = \frac{a(s - k_1)(s - k_2) \cdots (s - k_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}
\]

\[
H(s) = \frac{b(s - k_1)}{(s - p_1)(s - p_2)}
\]
Model Order Reduction

- Moment matching-based

\[ \hat{H}(s) = \sum_{i=1}^{q} \frac{r_i}{s-p_i} \quad \Leftrightarrow \quad \hat{h}(t) = \sum_{i=1}^{q} r_i e^{p_i t} \]

\[ \dot{x} = Ax + bu, \quad sX = AX + bU, \]
\[ y = c^T x \quad Y = c^T X \]
\[ H(s) = c^T (sI - A)^{-1} b = \sum_{i=0}^{\infty} m_i s^i \]
\[ m_i = -c^T A^{-i+1} b, \]
\[ m_{i+1} = A^{-1} m_i \]
**Power Distribution Estimation**

- **Power distribution estimation of interconnect**
  - Given a linear(ized) RLC circuits
  - Find power consumption of each resistor branch of interconnect

\[
E = R \int_0^T j^2(t) \, dt
\]

\[
J(s) = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}
\]

\[
j(t) = r_1 e^{p_1 t} + r_2 e^{p_2 t}
\]

Model order reduction
**Definition of problem**

- Given a reduced-order model of current at each resistor branch: $J(s) = \sum_{i=1}^{q} \frac{r_i}{s - p_i}$
- Derive: $E = R \int_{0}^{\infty} j^2(t) dt$

**Theorem 1**

- If the Laplace transform of a time-domain signal $j(t)$, denoted by $J(s)$, has $q$ singularities in the left half of the $s$-plane,

$$\int_{0}^{\infty} j^2(t) dt = \sum_{i=1}^{q} r_i$$

$r_i$: residue of $J(-s)J(s)$ at the singularity of $J(s)$
Power Distribution Estimation

- Sketch of proof

\[ \int_0^\infty j^2(t) dt = \left[ \int_0^\infty j^2(t) e^{-st} dt \right]_{s=0} \]

\[ = \left[ \mathcal{L}\{ j(t) j(t) \} \right]_{s=0} \]

\[ = \left[ \frac{1}{2\pi i} \lim_{T \to \infty} \int_{\gamma-iT}^{\gamma+iT} J(s-\omega) J(\omega) d\omega \right]_{s=0} \]

\[ = \frac{1}{2\pi i} \lim_{T \to \infty} \int_{\gamma-iT}^{\gamma+iT} J(-\omega) J(\omega) d\omega \]

\[ = \sum_{\text{residue of } J(-s) J(s)} \text{residue of } J(s) \]
**Power Distribution Estimation**

- **Example**

\[
J(s) = \frac{s + 3}{(s + 1)^2} = \frac{2}{(s + 1)^2} + \frac{1}{s + 1} \quad \leftrightarrow \quad j(t) = 2te^{-t} + e^{-t}
\]

\[
J(-s)J(s) = \frac{(-s + 3)(s + 3)}{(s - 1)^2 (s + 1)^2} = \frac{2}{(s - 1)^2} - \frac{5}{2} \frac{2}{s - 1} + \frac{2}{(s + 1)^2} + \frac{5}{2} \frac{1}{s + 1}
\]

\[
\int_0^\infty j^2(t)dt = \int_0^\infty (2te^{-t} + e^{-t})^2 dt = \frac{5}{2}
\]
Theorem 2

If the Laplace transform of a time-domain signal \( j(t) \), denoted by \( J(s) \), has \( q \) simple poles in the left half of the s-plane,

\[
\int_0^\infty j^2(t) dt = \sum_{i=1}^{q} r_i J(-p_i)
\]

\( r_i \): residue of \( J(s) \) at the pole \( p_i \) of \( J(s) \).
Power Distribution Estimation

- Example

\[
J(s) = \frac{3s + 5}{s^2 + 3s + 2} = \frac{2}{s + 1} + \frac{1}{s + 2} \quad \leftrightarrow \quad j(t) = 2e^{-t} + e^{-2t}
\]

\[
r_1 J(-p_1) + r_2 J(-p_2) = 2\left(1 + \frac{1}{3}\right) + 1\left(\frac{2}{3} + \frac{1}{4}\right) = \frac{43}{12}
\]

\[
\int_0^\infty j^2(t)dt = \int_0^\infty (2e^{-t} + e^{-2t})^2 dt = \frac{43}{12}
\]
Experimental Results

- **Prototype tool**
  - SPICE-in and power-out
  - Moment matching-based model order reduction

- **Estimation accuracy**
  - Source of error: area under the square of $j(t)$
  - Comparison with SPICE

\[ E = R \int_{0}^{T} j^2(t) \, dt \]
**Experimental Results**

- **Numerical example**

![Circuit Diagram]

<table>
<thead>
<tr>
<th>Resistor</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
<th>R6</th>
<th>R7</th>
<th>R8</th>
<th>R9</th>
<th>R10</th>
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<th>Max. error</th>
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<tbody>
<tr>
<td>SPICE</td>
<td>5.12</td>
<td>8.42</td>
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<td>0.24</td>
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Experimental Results

**Numerical example**

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Experimental Results

- **1-pole approximation**
  - Area under $j(t)$ and $\hat{j}(t)$ is the same
  - Area under $j^2(t)$ and $\hat{j}^2(t)$ depends on peakness and skewness of $j(t)$
Experimental Results

- Randomly-generated circuits
Driver Modeling

- Verify simple linear region resistance approximation for power distribution estimation
- Well below 10% both for max and avg error
Conclusion

- Power distribution is important for deep submicron interconnects
- Establish theoretical background for power distribution analysis of VLSI interconnects
- Develop and verify a simple driver model
- Future work
  - Fast yet accurate method
  - Investigation of accurate driver model